



How a Pensioner Should Invest, Consume, Annuitise and Bequeath

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Introduction

Consumption and portfolio optimisation during retirement has not received as much attention in financial research as optimisation prior to retirement. However, retirement planning is becoming more and more relevant for several reasons: rising conditional life expectancies, growing number of defined contribution pension plans in many countries and continuing wealth concentration among pensioners. In addition to the aforementioned statistical reasons there is of course academic interest in a good understanding of the end of the life cycle because this can be used as an input to economic models focusing on the labour phase.

The present article attempts to give a brief, non-technical summary of Schiess (2008) who studies the optimisation problem of a pensioner in a continuous time model. The agent of interest is a pensioner in the sense that he does not receive any stochastic labour income. Basically, a pensioner in the mentioned sense faces two main sources of uncertainty: longevity risk and investment risk. Consequently, we introduce a financial and an annuity market¹. On the one hand, the pensioner has to find the optimal time to annuitise his wealth (optimal stopping part). On the other hand, he should consume and invest optimally in the pre-annuitisation phase (optimal control part). Furthermore, these two problems are tightly connected through wealth. Thus, we solve a combined optimal stopping and optimal control problem (COSOCP). However, a pensioner derives utility not only from a stream of consumption or an annuity but also from bequeathing wealth to his heirs. The important inclusion of a bequest motive is the main difference between us and related literature as the two interesting articles of Stabile (2006) and Milevsky *et al.* (2007).

The Importance of a Bequest Motive

To keep the mentioned COSOCP tractable, we assume an all-or-nothing framework. This means that the pensioner under investigation is allowed to annuitise his entire remaining wealth at one (future) point in time (stopping time).² Furthermore, we assume power and subsistence level utility functions and the exponential mortality law. In the no-bequest case, he finds that the optimal annuitisation time is now-or-never: depending on the model parameters, the pensioner either annuitises now or never. However, we obtain an absurd strong tendency for the annuity market: it is optimal for the pensioner to annuitise his wealth immediately in almost all realistic situations. There are only very few situations where the pensioner chooses to stay in the financial market. This is *ceteris paribus* the case when one assumes a high enough Sharpe ratio (attractive financial market) and/or a low enough subjective life expectancy

¹ The annuitisation decision is modelled as irreversible due to adverse selection issues.

² In contrast, Milevsky *et al.* (2007) allows that the pensioner annuitises any fraction of his wealth anytime. However, they do not allow for a bequest motive.

(pensioner's perception towards his life expectancy) relative to the objective life expectancy (insurer's perception towards the pensioner's life expectancy) which implies an unattractive annuity market. This absurd strong tendency for the annuity market is absolutely inconsistent with the pensioners' decisions observed in reality.³ However, the obtained tendency is not surprising as we have not taken into account yet that people tend to leave a substantial part of their wealth at death. Indeed, things change dramatically if we introduce a bequest motive, since annuitisation is in clear conflict with bequests. Technically, the COSOCP with a bequest motive is much more involved.⁴ We find that the absurd strong tendency for the annuity market converts into a slight tendency for the financial market with the important inclusion of a bequest motive. Moreover, the annuitisation decision is normally not of the now-or-never type anymore but becomes wealth-dependent: it is optimal for the pensioner to annuitise as soon as his wealth level falls below some threshold. The dependence of this threshold on the model parameters is very natural. For instance, a higher level of priorly existing life insurance leads to a higher threshold and therefore increases the attractiveness of the annuity market. This is intuitive because the pensioner only bequeathes prior life insurance after annuitisation. On the other hand, a stronger bequest motive decreases the threshold and therefore decreases the attractiveness of the annuity market.

Longevity Risk Matters Very Much

Another main result concerns the relevance of longevity risk. Clearly, annuitisation is in conflict with bequests, as there is only prior life insurance that will be bequeathed after annuitisation. But even in the bequest-case there are many realistic situations where the pensioner chooses to annuitise his wealth. Thus, there are many realistic parameter settings (especially for rather risk-averse pensioners) where the pensioner is willing to eliminate his longevity risk by annuitising his wealth. Hence, the pensioner often prefers to guarantee for himself the consumption of the annuity during his remaining lifetime to the alternative of staying in the financial market which gives him the chance of bequeathing more than prior life insurance to his heirs but also the risk of outliving his assets.⁵ This highlights the importance of longevity risk and, in combination with the intergenerational risk transfer argument discussed in Baumann *et al.* (2008), it provides a legitimation for pension funds. Furthermore, this result is in line with the empirical observations in the Swiss pension funds that the majority of the insured prefer the lifelong pension (which at least partially⁶ eliminates their longevity risk) to the capital option.⁷ Lastly, the possibility to annuitise his wealth naturally affects the pensioner's consumption and investment rule even if he chooses to (at least for the moment) stay in the financial market. We show that the additional option of the annuity market leads to heavy consumption smoothing which is in contrast to the constant Merton consumption fraction. Moreover, it makes the pensioner more aggressive compared to the Merton investor for whom no annuity market exists at all. Figures 1 and 2 display the pensioner's wealth-dependent optimal consumption and investment rules for different values of the bequest motive⁸ η .

³ See for instance Milevsky *et al.* (2007).

⁴ Schiess (2008) uses duality arguments to simplify the optimisation problem and develops an iterative algorithm to solve the resulting problem.

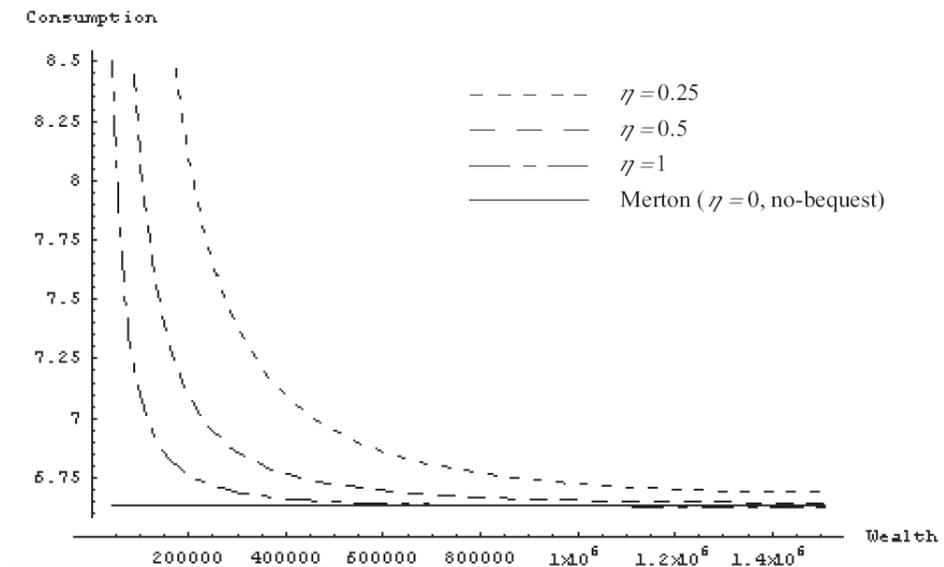
⁵ Furthermore, the pensioner of course keeps the flexibility of adjusting his consumption level and his investment strategy by staying in the financial market.

⁶ In contrast to the model studied by Schiess (2008), an individual of course has other wealth components besides the second pillar in reality.

⁷ The interested reader is referred to Büttler *et al.* (2007).

⁸ Technically, η is simply the coefficient of all bequest utility terms in the indirect utility function. The case $\eta = 0$ therefore corresponds to the no-bequest case while $\eta = 1$ means that the pensioner is altruistic in some sense.

Figure 1: Optimal consumption as a fraction of wealth for different values of the bequest motive η assuming prior life insurance net of subsistence level of bequests of 500, a relative risk aversion level of 2, an identical subjective and objective life expectancy of 20 years, an identical subjective and objective discount parameter of 0.035 and finally, a drift parameter of 0.08 and a diffusion parameter of 0.2 for the evolution of the risky asset (geometric Brownian motion).

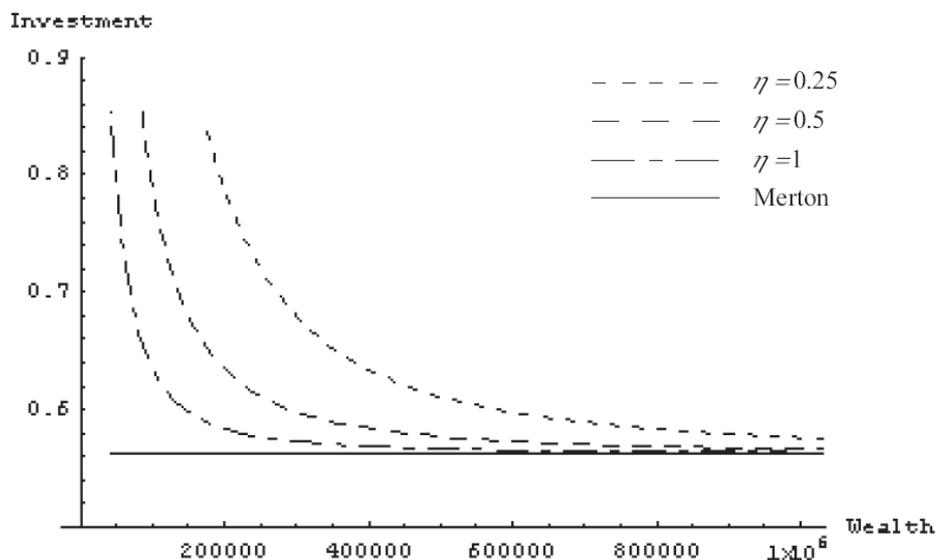


In Figure 2 we exhibit the Merton investment rule (fraction of wealth invested in the risky asset) for comparison. This constant investment rule (straight line) is naturally independent of the bequest parameter, as we use identical relative risk aversion for all utility functions. Specifically, the Merton

$$\frac{1}{\gamma} \frac{\mu - r}{\sigma^2} = \frac{1}{2} \frac{0.08 - 0.035}{0.2^2} = 0.5625$$

investment strategy is given by

Figure 2: Optimal investment in the risky asset for different values of the bequest motive η assuming prior life insurance net of subsistence level of bequests of 500, a relative risk aversion level of 2, an identical subjective and objective life expectancy of 20 years, an identical subjective and objective discount parameter of 0.035 and finally, a drift parameter of 0.08 and a diffusion parameter of 0.2 for the evolution of the risky asset (geometric Brownian motion).



Main Conclusions

The task of finding optimal consumption, asset allocation and the optimal annuitisation time for a pensioner leads to a combined optimal stopping and optimal control problem (COSOCP). In the no-bequest case it is optimal for the pensioner to annuitise immediately or never depending on the parameters of the model. However, there is an absurd strong tendency for the annuity market which can be eliminated with the inclusion of a bequest motive. Allowing for a bequest motive and consequently, for prior life insurance and a subsistence level of bequests, makes the optimisation problem technically quite involved and normally leads to a wealth-dependent annuitisation decision rule. The pensioner annuitises as soon as his wealth level falls below some threshold which exhibits a very natural parameter dependence. The main result is clearly that the essential inclusion of a bequest motive turns the very strong tendency for the annuity market into a slight tendency for the financial market. But even in the bequest case there are many realistic situations where the pensioner chooses the annuity market. This highlights the importance of longevity risk and, in combination with the intergenerational risk transfer argument discussed in Baumann *et al.* (2008), it provides a legitimation for pension funds.

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